

Planckian Birth of the Quantum de Sitter Universe

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Abstract

We show that the quantum universe emerging from a nonperturbative, Lorentzian sum-over-geometries can be described with high accuracy by a four-dimensional de Sitter spacetime. By a scaling analysis involving Newton's constant, we establish that the linear size of the quantum universes under study is in between 17 and 28 Planck lengths. Somewhat surprisingly, the measured quantum fluctuations around the de Sitter universe in this regime are to good approximation still describable semiclassically. The numerical evidence presented comes from a regularization of quantum gravity in terms of causal dynamical triangulations.

1 Introduction

To show that the physical spacetime surrounding us can be *derived* from some fundamental, quantum-dynamical principle is one of the holy grails of theoretical physics. The fact that this goal has been eluding us for the better part of the last half century could be taken as an indication that we have not as yet gone far enough in postulating new, exotic ingredients and inventing radically new construction principles governing physics at the relevant, ultra-high Planckian energy scale. – In this letter, we add to previous evidence that such a conclusion may be premature.

Our results are obtained in the framework of Lorentzian simplicial quantum gravity, based on the concept of causal dynamical triangulations (CDT). While referring to [1, 2, 3] for details, briefly, it defines a nonperturbative way of doing the sum over four-geometries, assembled from triangular building blocks such that only causal spacetime histories are included. To perform the actual summation, one rotates them to spacetimes of Euclidean signature. The building blocks are four-simplices characterized by a cut-off a , the side length of the simplices. The continuum limit of the regularized path integral corresponds to the limit $a \rightarrow 0$, possibly accompanied by a readjustment of bare coupling constants, and such that the physics stays invariant. The challenge of a quantum field theory of gravity is to find a theory which behaves in this way, and suitable observables to test it.

How can we judge whether CDT can be taken seriously as a regularized quantum field theory of gravity? Our knowledge of the physical world suggests that a viable theory should *generate* a ‘background geometry’ with positive cosmological constant, superposed with small quantum fluctuations. The challenge is to obtain this from a background-independent formulation where no background spacetime is put in by hand. We have earlier provided indirect evidence for such a scenario [4, 5]. Here, we present new computer simulations which confirm this picture much more directly, by establishing the de Sitter nature of the background spacetime, quantifying the fluctuations around it, and setting a scale for the universes we are dealing with.

2 Macroscopic de Sitter universe

By construction, each path integral geometry (Euclideanized by the analytic continuation mentioned above) is obtained by gluing together four-simplices in a way that respects a global foliation in discrete proper time t . Accordingly, each four-simplex will have “time-like” and space-like links of length a , our short-distance cutoff. Spatial slices are compact, three-dimensional manifolds of topology S^3 , glued from space-like tetrahedra. During simulations we fix the time direction to a given discrete length T_{tot} , and for reasons of convenience take it to be periodic

(with sufficiently long period so as to not affect results). The bare action used is the so-called Regge action, a geometric realization of the Einstein-Hilbert action on piecewise linear geometries.

A Monte Carlo (MC) simulation of the path integral – for computer-technical reasons performed at (almost) fixed four-volume – will generate a sequence of spacetime histories, represented by triangulations obeying causal gluing rules. The key result of CDT consisted in demonstrating that in a certain range of bare coupling constants¹ this leads to the *emergence of a four-dimensional universe of well-defined temporal and spatial extension* [4]. Here, we will analyze the geometric nature of this universe in more detail.

Plotting the ‘shape’ of MC-generated path integral configurations in the form of a time-dependent spatial three-volume $V_3(t)$, one typically obtains ‘blob’-like extended geometries of time extension $T \sim \sqrt[4]{V_4}$. Individual such configurations are of course not observable, in the same way that individual particle paths in a standard path integral are unphysical. There, one obtains the average particle trajectory by taking the average over paths in the path integral (with a weight dictated by the classical action), which agrees with the classical particle trajectory up to corrections of order \hbar . Doing this, for instance by MC simulations, yields information about both the average trajectory and the size of the quantum fluctuations, which in general will also be of order \hbar .

We will follow the same procedure for our gravitational histories. Averaging over many different computer-generated configurations will produce the background geometry around which the quantum universe fluctuates. Since the “centre of gravity” of our universes performs a random walk along the time direction, we take the average $\langle V_3(t) \rangle$ by identifying in each history the peak of the volume distribution with $t = 0$. (T_{tot} is always chosen much larger than the time extension T of the universes.) The results of measuring the average discrete spatial size of the universe at various discrete times i are illustrated in Fig. 1 and can be neatly summarized by the formula

$$N_3^{cl}(i) = \frac{3}{4} \frac{N_4}{s_0 N_4^{1/4}} \cos^3 \left(\frac{i}{s_0 N_4^{1/4}} \right), \quad s_0 \approx 0.59, \quad (1)$$

where $N_3(i)$ denotes the number of three-simplices in the spatial slice at discretized time i and N_4 the total number of four-simplices in the entire universe.

We have verified relation (1) for N_4 ranging from 45.500 to 362.000. The overlap of curves for different N_4 according to (1) constitutes a beautiful example of finite-size scaling [6]. Eq. (1) shows that spatial volumes scale according to

¹Other choices of bare couplings can lead to different and degenerate geometries [1].

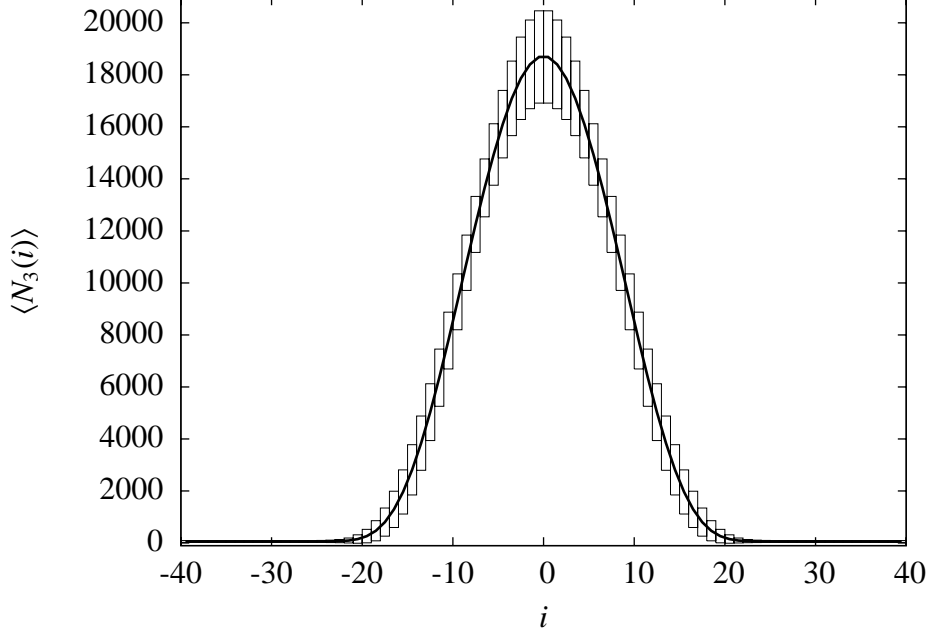


Figure 1: Background geometry $\langle N_3(i) \rangle$: MC measurements (for fixed $N_4 = 362.000$) and best fit (1) yield indistinguishable curves at given plot resolution. The bars indicate the average size of quantum fluctuations.

$N_4^{3/4}$ and time intervals according to $N_4^{1/4}$, as one would expect for a genuinely *four*-dimensional spacetime. Translating (1) to a continuum notation leads to

$$\sqrt{g_{tt}} V_3^{cl}(t) = V_4 \frac{3}{4B} \cos^3 \left(\frac{t}{B} \right), \quad (2)$$

where we have made the identifications

$$V_4 = a^4 C_4 N_4, \quad \sqrt{g_{tt}} V_3 = a^3 C_4 N_3, \quad t_i = a i. \quad (3)$$

In (3), $C_4 = \sqrt{5}/96$ from the discrete four-simplex volume [2], and $\sqrt{g_{tt}}$ is the constant proportionality factor between the time t and genuine continuum proper time τ , $\tau = \sqrt{g_{tt}} t$. Writing $V_4 = 8\pi^2 R^4/3$, and $\sqrt{g_{tt}} = R/B$, eq. (2) is seen to describe a Euclidean *de Sitter universe* (a four-sphere, the maximally symmetric space for positive cosmological constant) as our searched-for, dynamically generated background geometry! In the parametrization of (2) this is the classical solution to the action

$$S = \frac{1}{24\pi G} \int dt \sqrt{g_{tt}} \left(\frac{g^{tt} \dot{V}_3^2(t)}{V_3(t)} + k_2 V_3^{1/3} - l V_3(t) \right), \quad (4)$$

where $k_2 = 9(2\pi^2)^{2/3}$ and l is a Lagrange multiplier, fixed by requiring that the total four-volume be V_4 , $\int dt \sqrt{g_{tt}} V_3(t) = V_4$. Up to an overall sign, this is precisely the Einstein-Hilbert action for the scale factor $a(t)$ of a homogeneous, isotropic universe (rewritten in terms of $V_3(t) = 2\pi^2 a(t)^3$), although we of course never put any such simplifying symmetry assumptions into the CDT model. The discretized, dimensionless version of (4) is

$$S_{discr} = k_1 \sum_i \left(\frac{(N_3(i+1) - N_3(i))^2}{N_3(i)} + \tilde{k}_2 N_3^{1/3} \right), \quad (5)$$

where $\tilde{k}_2 \propto k_2$. The identifications (3) lead to a naïve continuum limit of the discretized action S_{discr} with

$$G = \frac{a^2}{k_1} \frac{C_4}{24\pi g_{tt}}. \quad (6)$$

Our next aim will be to determine the coefficient k_1 in front of the effective action (5) from the computer simulations. Because of relation (6), this will give us an estimate of the gravitational coupling constant G in terms of the lattice spacing a . Since in our units the Planck length is $\ell_{Pl} = \sqrt{G}$, *this will set a physical length scale for lattice quantities*. While the classical solution (2) does not provide information about the numerical value of k_1 in front of (4), a saddle point calculation shows that the fluctuations $\delta V_3(t) := V_3(t) - V_3^{cl}(t)$ around $V_3^{cl}(t) = \langle V_3(t) \rangle$ will be of the order

$$\langle (\delta V_3)^2 \rangle \sim G V_4. \quad (7)$$

Therefore, *if* it is true that also the fluctuations $\delta N_3(i) := N_3(i) - N_3^{cl}(i)$ of our model are well described by the mini-superspace action (4), we can simply determine k_1 from measuring their correlator.

3 Fluctuations around de Sitter space

Having demonstrated that the action (4) gives a perfect description of the measured $V_3^{cl}(t)$, we will now show that *it also describes the observed quantum fluctuations around de Sitter space*, defined by the correlator

$$C(t, t') = \langle \delta V_3(t) \delta V_3(t') \rangle. \quad (8)$$

Our first observation from the data is that the discretized version of (8) scales with the four-volume according to

$$C_{N_4}(i, i') = \langle \delta N_3(i) \delta N_3(i') \rangle = N_4 F\left(i/N_4^{1/4}, i'/N_4^{1/4}\right), \quad (9)$$

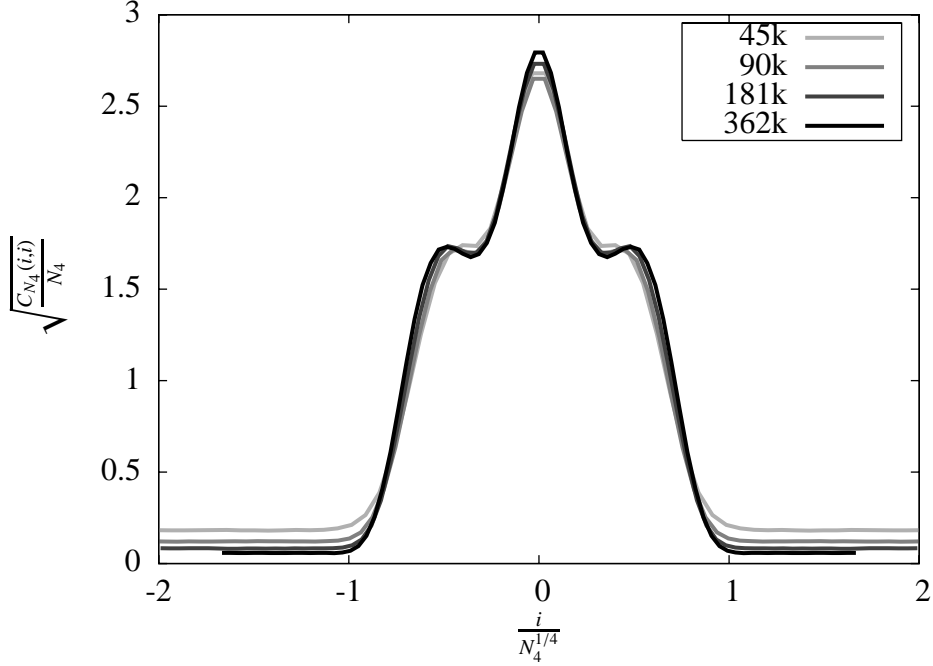


Figure 2: Analyzing the quantum fluctuations of Fig. 1: diagonal entries $F(t, t)^{1/2}$ of the scaling function F from (9), for $N_4 = 45.500, 91.000, 181.000$ and 362.000 .

where F is a universal scaling function of order a^0 . This is illustrated by Fig. 2 for $C_{N_4}^{1/2}(i, i)$, corresponding precisely to the fluctuations $\langle (\delta V_3(t))^2 \rangle^{1/2}$ of Fig. 1. This scaling implies that the action (5) can only describe the fluctuations measured for different N_4 if k_1 is independent of N_4 , thus confirming the scaling behaviour $G \sim a^2$ anticipated in (6).

To demonstrate that $F(t, t')$ is indeed described by the effective actions (4), (5), let us for convenience adopt a continuum language and compute its expected behaviour. Expanding (4) around the classical solution as $V_3(t) = V_3^{cl}(t) + x(t)$, the quadratic fluctuations are given by

$$\begin{aligned} \langle x(t)x(t') \rangle &= \int \mathcal{D}x(s) x(t)x(t') e^{-\frac{1}{2} \iint ds ds' x(s) M(s, s') x(s')} \\ &= M^{-1}(t, t'), \end{aligned} \quad (10)$$

where $\mathcal{D}x(s)$ is the normalized measure and the quadratic form $M(t, t')$ is determined by expanding the effective action S to second order in $x(t)$,

$$S(V_3) = S(V_3^{cl}) + \frac{1}{18\pi G} \frac{B}{V_4} \int dt x(t) \hat{H} x(t). \quad (11)$$

In (11), \hat{H} denotes the Hermitian operator

$$\hat{H} = -\frac{d}{dt} \frac{1}{\cos^3(t/B)} \frac{d}{dt} - \frac{4}{B^2 \cos^5(t/B)}, \quad (12)$$

which must be diagonalized under the constraint that $\int dt \sqrt{g_{tt}} x(t) = 0$, since V_4 is kept constant.

Let $e_n(t)$ be the eigenfunctions of the quadratic form given by (11) with the volume constraint enforced, ordered according to increasing eigenvalues l_n . If this cosmological continuum model were to give the correct description of the computer-generated universe, the matrix

$$M^{-1}(t, t') = \sum_{n=1}^{\infty} \frac{e_n(t) e_n(t')}{l_n}. \quad (13)$$

should be proportional to the measured correlator $C(t, t')$. Fig. 3 shows the highest eigenfunction calculated from the data, the matrix $C(t, t')$, and the corresponding lowest eigenfunction calculated from the effective action, the matrix $M(t, t')$. The agreement is very good, in particular taking into account that no parameter is adjusted in the action (we simply take $B = s_0 N_4^{1/4}$ in (2) and (11), i.e. $14.47a$ for $N_4 = 362.000$). One can also compare the data and the matrix $M^{-1}(t, t')$ calculated from (13) directly. The agreement is again good, although less spectacular than in Fig. 3. Awaiting publication of a full analysis of the data [6], suffice it to say that within measuring accuracy the fluctuations of the spatial volume are described by the mini-superspace action (4). This enables us to numerically estimate $k_1 \approx 0.016$, which according to (6) leads to $G \approx 0.22a^2$, or $\ell_{Pl} \approx 0.47a$. In other words, *the linear size of the quantum de Sitter universes studied here lies in the range of 17-28 Planck lengths.*

4 Discussion

The CDT model of quantum gravity is extremely simple, namely, the path integral over the class of causal geometries with a global time foliation. In order to perform this summation explicitly, we introduce a grid of piecewise linear geometries, much in the same way as when defining the path integral in quantum mechanics. Next, we rotate each of these geometries to Euclidean signature and use as bare action the Einstein-Hilbert action ² in Regge form. Nothing else is put in.

The resulting superposition exhibits scaling behaviour as function of the four-volume, and we observe the appearance of a well-defined average geometry, that

²Of course, the full, effective action, including measure contributions, will contain all higher-derivative terms.

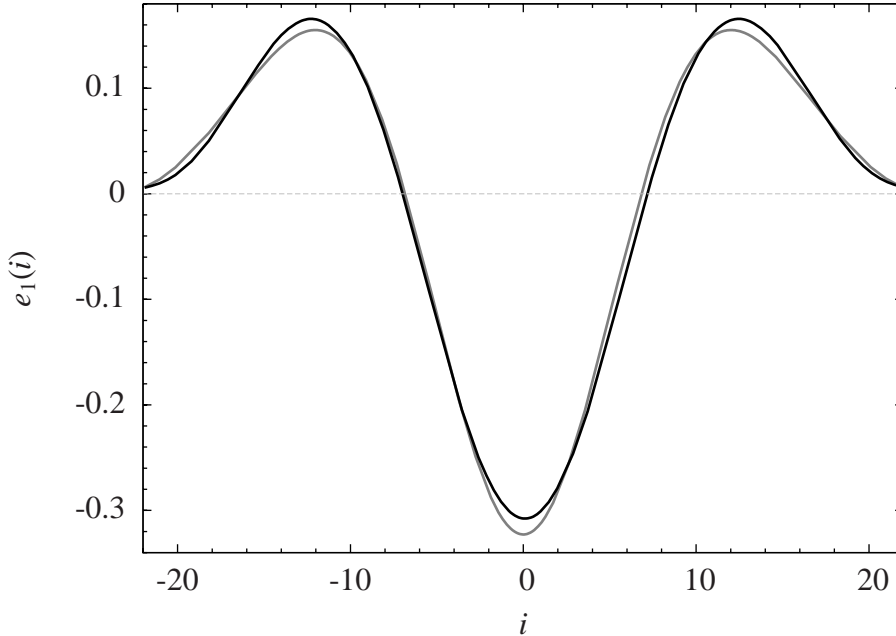


Figure 3: Comparing the highest eigenvector of $C(t, t')$ and the lowest eigenvector of $M^{-1}(t, t')$.

of de Sitter space. We are definitely in a quantum regime, since the fluctuations around de Sitter space are sizeable, as can be seen in Fig. 1. Both the average geometry and the quantum fluctuations are well described by the mini-superspace action (4). Unlike in standard cosmological treatments, this description is the *outcome* of a nonperturbative evaluation of the *full* path integral, with everything but the scale factor (equivalently, $V_3(t)$) summed over. Measuring the correlations of the fluctuations in the computer simulation enabled us to determine the continuum gravitational coupling constant to $G \approx 0.22a^2$, thereby introducing an absolute physical length scale. Within measuring accuracy, our de Sitter universes (with volumes in the range 22.000-173.000 ℓ_{Pl}^4) are seen to behave perfectly semiclassically.

Can we study smaller universes, which are themselves of Planck size? Taking the coupling G as a true measure of the gravitational coupling constant, the simplest way is as follows. We are free to vary N_4 and the bare gravitational coupling constant g_0 of the Regge action (see [1] for further details on the bare coupling constants), with the effective constant k_1 a function of g_0 . If we adjusted g_0 such that in the limit $N_4 \rightarrow \infty$ both

$$V_4 \sim N_4 a^4 \quad \text{and} \quad G \sim a^2/k_1(g_0) \quad (14)$$

remained constant (i.e. $k_1(g_0) \sim 1/\sqrt{N_4}$), we would eventually penetrate into the

(sub-)Planckian regime. Since we have already seen deviations from classicality at short scales [1], we would expect the canonical scaling of G to change there, or, stated differently, the simple effective action (4) to be no longer valid. Renormalization group methods have produced predictions for the scaling violations of G in the context of asymptotic safety [7], which in principle we should be able to test. In this context it would be ideal to have an observable with an associated correlation length that could be kept constant when expressed in terms of $V_4^{1/4}$. A further step will be to include matter in the model and verify directly that G can indeed be interpreted as Newton's constant, perhaps along the lines pursued earlier in Euclidean quantum gravity [8]. All of these issues are currently under investigation.

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